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LU-Decomposition of a Noncommutative Linear System and Jacobi Polynomials

We obtain the LU-decomposition of a non commutative linear system of equations that, in the rank one case, characterizes the image of the Lepowsky homomorphism $U(\mathfrak{g})^K \to U(\mathfrak{k})^M \otimes U(\mathfrak{a})$. Although this system can not be expressed as a single matrix equation with coefficients in $U(\mathfrak{k})$, it turns out that obtaining a triangular system equivalent to it, can be reduced to obtaining the LU-decomposition of a matrix $\widetilde{M}_0$ with entries in a polynomial algebra. We prove that both the L-part and U-part of $\widetilde{M}_0$ are expressed in terms of Jacobi polynomials. Moreover, each entry of the L-part of $\widetilde{M}_0$ and of its inverse is given by a single ultraspherical Jacobi polynomial. This fact yields a biorthogonality relation between the ultraspherical Jacobi polynomials.

Keywords: Noncommutative LU-factorization, Jacobi polynomials, K-invariants in the enveloping algebra of g, Lepowsky homomorphism.

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