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Diamond Representations for Rank Two Semisimple Lie Algebras

The present work is a part of a larger program to construct explicit combinatorial models for the (indecomposable) regular representation of the nilpotent factor N in the Iwasawa decomposition of a semisimple Lie algebra $\mathfrak{g}$, using the restrictions to N of the simple finite dimensional modules of $\mathfrak{g}$. Such a description was given by D. Arnal, N. Bel Baraka and N.-J. Wildberger [*Diamond representations of* $\mathfrak{sl}(n)$, International Journal of Algebra and Computation 13 (2006) 381–429] for the case $\mathfrak{g} = \mathfrak{sl}(n)$. Here, we perform the same construction for the rank 2 semisimple Lie algebras (of type $A_1 \times A_1$, A_2, C_2 and G_2). The algebra $\mathbb{C}[N]$ of polynomial functions on N is a quotient, called the reduced shape algebra, of the shape algebra for $\mathfrak{g}$. Bases for the shape algebra are known, for instance the so-called semistandard Young tableaux [see L.-W. Alverson, R.-G. Donnelly, S.-J. Lewis, M. McClard, R. Pervine, R.-A. Proctor, and N.-J. Wildberger, *Distributive lattice defined for representations of rank two semisimple Lie algebras*, ArXiv 0707.2421 v 1 (2007) 1–33] give an explicit basis. We select among the semistandard tableaux, the so-called quasistandard ones which define a kind basis for the reduced shape algebra.

Keywords: Rank two semisimple Lie algebras, representations, Young tableaux.

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