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Higher Arf Functions and Moduli Space of Higher Spin Surfaces

We describe all connected components of the space of pairs (P, s), where P is a hyperbolic Riemann surface with finitely generated fundamental group and s is an m-spin structure on P. We prove that any connected component is homeomorphic to a quotient of $\mathbb{R}^d$ by a discrete group.

Our method is based on a description of an m-spin structure by an m-Arf function, that is a map $\sigma\colon \pi_1(P, p) \to \mathbb{Z}/m\mathbb{Z}$ with certain geometric properties. We prove that the set of all m-Arf functions has a structure of an affine space associated with $H_1(P, \mathbb{Z}/m\mathbb{Z})$. We describe the orbits of m-Arf functions under the action of the group of homotopy classes of surface autohomeomorphisms. Natural topological invariants of an orbit are the unordered set of values of the m-Arf functions on the punctures and the unordered set of values on the m-Arf-function on the holes. We prove that for $g > 1$ the space of m-Arf functions with prescribed genus and prescribed (unordered) sets of values on punctures and holes is either connected or has two connected components distinguished by the Arf invariant $\delta \in \{0, 1\}$. Results for $g = 1$ are also given.

Keywords: Higher spin surfaces, Arf functions, lifts of Fuchsian groups.

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