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Buildings of Classical Groups and Centralizers of Lie Algebra Elements

Let $F_\circ$ be a non-archimedean locally compact field of residual characteristic not 2. Let G be a classical group over $F_\circ$ (with no quaternionic algebra involved) which is not a general linear group. Let β be an element of the Lie algebra $\mathfrak{g}$ of G that we assume semisimple for simplicity. Let H be the centralizer of β in G and $\mathfrak{h}$ its Lie algebra. Let I and I_β^1 denote the (enlarged) Bruhat-Tits buildings of G and H respectively. We prove that there is a natural set of maps $j_\beta \colon I_\beta^1 \to I$ which enjoy the following properties: they are affine, H-equivariant, map any apartment of I_β^1 into an apartment of I and are compatible with the Lie algebra filtrations of $\mathfrak{g}$ and $\mathfrak{h}$. In a particular case, where this set is reduced to one element, we prove that j_β is characterized by the last property in the list. We also prove a similar characterization result for the general linear group.

In this article, we work with Lie algebra filtrations defined by using lattice models of buildings. It is not clear that they coincide with the filtrations constructed by A. Moy and G. Prasad for a general reductive group. This fact is proved by B. Lemaire (see his article in this volume, pp. 29–54).

Keywords: Bruhat-Tits building, classical groups over p-adic fields, Moy and Prasad filtrations, functoriality of affine buildings.

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