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## Proper Actions on Corank-One Reductive Homogeneous Spaces

Let  $\mathbf{k}$  be a local field, G the set of  $\mathbf{k}$ -points of a connected semisimple algebraic  $\mathbf{k}$ -group  $\mathbf{G}$ , and H the set of  $\mathbf{k}$ -points of a connected reductive algebraic  $\mathbf{k}$ -subgroup  $\mathbf{H}$  of  $\mathbf{G}$  such that  $\operatorname{rank}_{\mathbf{k}}(\mathbf{H}) = \operatorname{rank}_{\mathbf{k}}(\mathbf{G}) - \mathbf{1}$ . We consider discrete subgroups  $\Gamma$  of G acting properly discontinuously on G/H and we examine their images under a Cartan projection  $\mu : G \to V^+$ , where  $V^+$  is a closed convex cone in a real finite-dimensional vector space. We show that if  $\Gamma$  is neither a torsion group nor a virtually cyclic group, then  $\mu(\Gamma)$  is almost entirely contained in one connected component of  $V^+ \setminus C_H$ , where  $C_H$  denotes the convex hull of  $\mu(H)$  in  $V^+$ . As an application, we describe all torsion-free discrete subgroups of  $G \times G$  acting properly discontinuously on G by left and right translation when  $\operatorname{rank}_{\mathbf{k}}(\mathbf{G}) = \mathbf{1}$ .

**Keywords**: Discrete subgroups of Lie groups, discrete subgroups of p-adic groups, reductive groups over local fields, properly discontinuous action, Cartan decomposition.

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