Journal of Lie Theory 18 (2008) 851–867

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A Satake Type Theorem for Super Automorphic Forms

The aim of this article is a Satake type theorem for super automorphic forms on a complex bounded symmetric super domain $\mathcal{B}$ of rank 1 with respect to a lattice Γ. 'Super' means: additional odd (anticommuting) coordinates on an ordinary complex bounded symmetric domain B (the so-called body of $\mathcal{B}$) of rank 1. Satake's theorem says that for large weight k all spaces

$$sM_k(\Gamma) \cap L_k^s(\Gamma\backslash\mathcal{B}),$$

$s \in [1, \infty]$ coincide, where $sM_k(\Gamma)$ denotes the space of super automorphic forms for Γ with respect to the weight k, and $L_k^s(\Gamma\backslash\mathcal{B})$ denotes the space of s-intergrable functions with respect to a certain measure on the quotient $\Gamma\backslash\mathcal{B}$ depending on k. So all these spaces are equal to the space $sS_k(\Gamma) := sM_k(\Gamma) \cap L_k^2(\Gamma\backslash\mathcal{B}$ of super cusp forms for Γ to the weight k.
As it is already well known for automorphic forms on ordinary complex bounded symmetric domains, we will give a proof of this theorem using an unbounded realization $\mathcal{H}$ of $\mathcal{B}$ and Fourier decomposition at the cusps of the quotient $\Gamma\backslash B$ mapped to ∞ via a partial Cayley transformation.

Keywords: Automorphic and cusp forms, complex bounded symmetric domains, super symmetry, semisimple Lie groups, unbounded realization of a complex bounded symmetric domain.

MSC: 11F55; 32C11