

A. Caine

Max-Planck-Institut für Mathematik, Postfach 7280, 53111 Bonn, Germany
caine@mpim-bonn.mpg.de

Compact Symmetric Spaces, Triangular Factorization, and Poisson Geometry

Let X be a simply connected compact Riemannian symmetric space, let U be the universal covering group of the identity component of the isometry group of X , and let \mathfrak{g} denote the complexification of the Lie algebra of U , $\mathfrak{g} = \mathfrak{u}^{\mathbb{C}}$. Each \mathfrak{u} -compatible triangular decomposition $\mathfrak{g} = \mathfrak{n}_- + \mathfrak{h} + \mathfrak{n}_+$ determines a Poisson Lie group structure π_U on U . The Evens-Lu construction produces a (U, π_U) -homogeneous Poisson structure on X . By choosing the basepoint in X appropriately, X is presented as U/K where K is the fixed point set of an involution which stabilizes the triangular decomposition of \mathfrak{g} . With this presentation, a connection is established between the symplectic foliation of the Evens-Lu Poisson structure and the Birkhoff decomposition of U/K . This is done through reinterpretation of results of Pickrell. Each symplectic leaf admits a natural torus action. It is shown that the action is Hamiltonian and the momentum map is computed using triangular factorization. Finally, local formulas for the Evens-Lu Poisson structure are displayed in several examples.

Keywords: Homogeneous poisson structures, symmetric spaces, momentum map.

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