

E. Dan-Cohen

Dept. of Mathematics, University of California, Berkeley, CA 94720, U.S.A.
edc@math.berkeley.edu

Borel Subalgebras of Root-Reductive Lie Algebras

This paper generalizes the classification of I. Dimitrov and I. Penkov [*Borel subalgebras of $\mathfrak{gl}(\infty)$* , Resenhas 6 (2004) 153–163] of Borel subalgebras of \mathfrak{gl}_∞ . Root-reductive Lie algebras are direct limits of finite-dimensional reductive Lie algebras along inclusions preserving the root spaces with respect to nested Cartan subalgebras. A Borel subalgebra of a root-reductive Lie algebra is by definition a maximal locally solvable subalgebra. The main general result of this paper is that a Borel subalgebra of an infinite-dimensional indecomposable root-reductive Lie algebra is the simultaneous stabilizer of a certain type of generalized flag in each of the standard representations.

For the three infinite-dimensional simple root-reductive Lie algebras more precise results are obtained. The map sending a maximal closed (isotropic) generalized flag in the standard representation to its stabilizer hits Borel subalgebras, yielding a bijection in the cases of \mathfrak{sl}_∞ and \mathfrak{sp}_∞ ; in the case of \mathfrak{so}_∞ the fibers are of size one and two. A description is given of a nice class of toral subalgebras contained in any Borel subalgebra. Finally, certain Borel subalgebras of a general root-reductive Lie algebra are seen to correspond bijectively with Borel subalgebras of the commutator subalgebra, which are understood in terms of the special cases.

Keywords: Locally finite Lie algebra, root-reductive Lie algebra, Borel subalgebra, maximal locally solvable subalgebra.

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