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K. Schmüdgen

Mathematisches Institut, Universität Leipzig, Johannisgasse 26, 04103 Leipzig, Germany
schmuedgen@math.uni-leipzig.de

E. Wagner

Facultad de Ciencias, Universidad de Colima, Bernal Díaz del Castillo 340, 28045 Colima, Mexico
Elmar.Wagner@math.uni-leipzig.de

Representations of Crossed Product Algebras of Podleś Quantum Spheres

Hilbert space representations of the crossed product $*$ -algebras of the Hopf $*$ -algebra $\mathcal{U}_q(\mathfrak{su}_2)$ and its module $*$ -algebras $\mathcal{O}(S_{\text{qr}}^2)$ of Podleś spheres are investigated and classified by describing the action of generators. The representations are analyzed within two approaches. It is shown that the Hopf $*$ -algebra $\mathcal{O}(\text{SU}_q(2))$ of the quantum group $\text{SU}_q(2)$ decomposes into an orthogonal sum of projective Hopf modules corresponding to irreducible integrable $*$ -representations of the crossed product algebras and that each irreducible integrable $*$ -representation appears with multiplicity one. The projections of these projective modules are computed. The decompositions of tensor products of irreducible integrable $*$ -representations with spin l representations of $\mathcal{U}_q(\mathfrak{su}_2)$ are given. The invariant state h on $\mathcal{O}(S_{\text{qr}}^2)$ is studied in detail. By passing to function algebras over the quantum spheres S_{qr}^2 , we give chart descriptions of quantum line bundles and describe the representations from the first approach by means of the second approach.

Keywords: Quantum groups, unbounded representations.

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