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Construction of Groups Associated to Lie- and to Leibniz-Algebras

We describe a method for associating to a Lie algebra $\mathfrak{g}$ over a ring $\mathbb{K}$ a sequence of groups $(G_n(\mathfrak{g}))_{n\in\mathbb{N}}$, which are *polynomial groups* in the sense that will be explained in Definition 5.1. Using a description of these groups by generators and relations, we prove the existence of an action of the symmetric group Σ_n by automorphisms. The subgroup of fixed points under this action, denoted by $J_n(\mathfrak{g})$, is still a polynomial group and we can form the projective limit $J_\infty(\mathfrak{g})$ of the sequence $(J_n(\mathfrak{g}))_{n\in\mathbb{N}}$. The formal group $J_\infty(\mathfrak{g})$ associated in this way to the Lie algebra $\mathfrak{g}$ may be seen as a generalisation of the formal group associated to a Lie algebra over a field of characteristic zero by the Campbell-Haussdorf formula.

Keywords: Lie algebra, Leibniz algebra, polynomial group, formal group, exponential map, Campbell-Haussdorf formula, dual numbers.

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