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Z. Reichstein

Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, Canada
reichst@math.ubc.ca

A. Vistoli

Dipartimento di Matematica, Università di Bologna, Piazza di Porta S. Donato 5, 40137
Bologna, Italy
vistoli@dm.unibo.it

Birational Isomorphisms between Twisted Group Actions

Let X be an algebraic variety with a generically free action of a connected algebraic group G . Given an automorphism $\phi: G \rightarrow G$, we will denote by X^ϕ the same variety X with the G -action given by $g: x \rightarrow \phi(g) \cdot x$. We construct examples of G -varieties X such that X and X^ϕ are not G -equivariantly isomorphic. The problem of whether or not such examples can exist in the case where X is a vector space with a generically free linear action, remains open. On the other hand, we prove that X and X^ϕ are always stably birationally isomorphic, i.e., $X \times \mathbb{A}^m$ and $X^\phi \times \mathbb{A}^m$ are G -equivariantly birationally isomorphic for a suitable $m \geq 0$.

Keywords: Group action, algebraic group, no-name lemma, birational isomorphism, central simple algebra, Galois cohomology.

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