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Superpfaffian

Let $V = V_0 \oplus V_1$ be a real finite dimensional supervector space provided with a nondegenerate antisymmetric even bilinear form B. Let $\mathfrak{spo}(V)$ be the Lie superalgebra of endomorphisms of V which preserve B. We consider $\mathfrak{spo}(V)$ as a supermanifold. We show that a choice of an orientation of V_1 and of a square root $\mathbf{i}$ of -1 determines a very interesting generalized function on the supermanifold $\mathfrak{spo}(V)$, the *superpfaffian*. When $V = V_1$, $\mathfrak{spo}(V)$ is the orthogonal Lie algebra $\mathfrak{so}(V_1)$, the superpfaffian is the usual Pfaffian, a square root of the determinant. When $V = V_0$, $\mathfrak{spo}(V)$ is the symplectic Lie algebra $\mathfrak{sp}(V_0)$, the superpfaffian is a constant multiple of the Fourier transform of one the two minimal nilpotent orbits in the dual of the Lie algebra $\mathfrak{sp}(V_0)$, and it is a square root of the inverse of the determinant in the open subset of invertible elements of $\mathfrak{spo}(V)$. In this article, we present the definition and some basic properties of the superpfaffian.

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