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Operator Kernels for Irreducible Unitary Representations of Solvable Exponential Lie Groups

Let G be a connected, simply connected, exponential solvable Lie group. The irreducible unitary representations of G may be obtained by the Kirillov-Bernat orbit method. Let $l \in \mathfrak{g}^*$, $\mathfrak{p}$ a Pukanszky polarization associated to l, $P = \exp \mathfrak{p}$, χ_l the corresponding character of P and $\pi_l = \mathrm{ind}_P^G \chi_l$ the associated unitary representation. We show through an example that not all the functions of $\mathcal{C}_c^\infty(G/P, G/P, \chi_l)$ ($\mathcal{C}^\infty$-functions with compact support on $G/P \times G/P$ satisfying a certain covariance condition) are kernel functions of some operator of the form $\pi_l(f)$, $f \in L^1(G)$, even if the polarization is well chosen. This contradicts a result of H. Leptin [J. Reine Angew. Math. 494 (1998) 1–34]). But if the polarization $\mathfrak{p}$ is an ideal of $\mathfrak{g}$, then the result of Leptin is true, the corresponding retract from $\mathcal{C}_c^\infty(G/P, G/P, \chi_l)$ into $L^1(G)$ exists and a construction algorithm of the function f may be indicated.

Keywords: Irreducible unitary representation, kernel of an operator, retract.

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