

© 2005 Heldermann Verlag
Journal of Lie Theory 15 (2005) 299–320

P. Ramacher

Humboldt-Universität, Institut für Reine Mathematik, Rudower Chaussee 25, 10099 Berlin,
Germany
ramacher@mathematik.hu-berlin.de

Analysis on Real Affine G -Varieties

We consider the action of a real linear algebraic group G on a smooth, real affine algebraic variety $M \subset \mathbb{R}^n$, and study the corresponding left regular representation of G on the Banach space $C_0(M)$ of continuous, complex valued functions on M vanishing at infinity. We show that the differential structure of this representation is already completely characterized by the action of the Lie algebra \mathfrak{g} of G on the dense subspace $\mathcal{P} = \mathbb{C}[M] \cdot e^{-r^2}$, where $\mathbb{C}[M]$ denotes the algebra of regular functions of M and r the distance function in \mathbb{R}^n . We prove that the elements of this subspace constitute analytic vectors of the considered representation, and by taking into account the algebraic structure of \mathcal{P} , we obtain G -invariant decompositions and discrete reducing series of $C_0(M)$. In case that G is reductive, K a maximal compact subgroup, \mathcal{P} turns out to be a (\mathfrak{g}, K) -module in the sense of Harish-Chandra and Lepowsky, and by taking suitable subquotients of \mathcal{P} , respectively $C_0(M)$, one gets admissible (\mathfrak{g}, K) -modules as well as K -finite Banach representations.

Keywords: G -varieties, Banach representations, real reductive groups, dense graph theorem, analytic elements, (\mathfrak{g}, K) -modules, reducing series.

MSC: 57S25; 22E45, 22E46, 22E47, 47D03