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Tits Geometry, Arithmetic Groups, and the Proof of a Conjecture of Siegel

Let $X = G/K$ be a Riemannian symmetric space of noncompact type and of rank ≥ 2. An irreducible, non-uniform lattice $\Gamma \subset G$ in the isometry group of X is arithmetic and gives rise to a locally symmetric space $V = \Gamma\backslash X$. Let $\pi : X \to V$ be the canonical projection. Reduction theory for arithmetic groups provides a dissection $V = \coprod_{i=1}^{k} \pi(X_i)$ with $\pi(X_0)$ compact and such that the restiction of π to X_i is injective for each i. In this paper we complete reduction theory by focusing on metric properties of the sets X_i. We detect subsets C_i of X_i ($\mathbb{Q}$–Weyl chambers) such that $\pi_{|C_i}$ is an isometry and such that C_i is a net in X_i. This result is then used to prove a conjecture of C.L. Siegel. We also show that V is quasi-isometric to the Euclidean cone over a finite simplicial complex and study the Tits geometry of V.