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Determination of the Topological Structure of an Orbifold by its Group of Orbifold Diffeomorphisms

We show that the topological structure of a compact, locally smooth orbifold is determined by its orbifold diffeomorphism group. Let $\mathrm{Diff}^r_{\mathrm{Orb}}(\mathcal{O})$ denote the C^r orbifold diffeomorphisms of an orbifold $\mathcal{O}$. Suppose that $\Phi\colon \mathrm{Diff}^r_{\mathrm{Orb}}(\mathcal{O}_1) \to \mathrm{Diff}^r_{\mathrm{Orb}}(\mathcal{O}_2)$ is a group isomorphism between the the orbifold diffeomorphism groups of two orbifolds $\mathcal{O}_1$ and $\mathcal{O}_2$. We show that Φ is induced by a homeomorphism $h\colon X_{\mathcal{O}_1} \to X_{\mathcal{O}_2}$, where $X_{\mathcal{O}}$ denotes the underlying topological space of $\mathcal{O}$. That is, $\Phi(f) = hfh^{-1}$ for all $f \in \mathrm{Diff}^r_{\mathrm{Orb}}(\mathcal{O}_1)$. Furthermore, if $r > 0$, then h is a C^r manifold diffeomorphism when restricted to the complement of the singular set of each stratum.