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A Leibniz Algebra Structure on the Second Tensor Product

For any Lie algebra $\mathfrak{g}$, the bracket

$$[x \otimes y, a \otimes b] := [x, [a, b]] \otimes y + x \otimes [y, [a, b]]$$

defines a Leibniz algebra structure on the vector space $\mathfrak{g} \otimes \mathfrak{g}$. We let $\mathfrak{g} \underline{\otimes} \mathfrak{g}$ be the maximal Lie algebra quotient of $\mathfrak{g} \otimes \mathfrak{g}$. We prove that this particular Lie algebra is an abelian extension of the Lie algebra version of the nonabelian tensor product $\mathfrak{g} \boxtimes \mathfrak{g}$ of R. Brown and J.-L. Loday [Topology 26 (1987) 311–335] constructed by G. J. Ellis [J. Pure Appl. Algebra 46 (1987) 111–115; Glasgow Math. J. 33 (1991) 101–120]. We compute this abelian extension and Leibniz homology of $\mathfrak{g} \otimes \mathfrak{g}$ in the case, when $\mathfrak{g}$ is a finite dimensional semi-simple Lie algebra over a field of characteristic zero.