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Characterization of the L^p-Range of the Poisson Transform in Hyperbolic Spaces $B(\mathbb{F}^n)$

The aim of this paper is to give, in a unified manner, the characterization of the L^p-range ($p \geq 2$) of the Poisson transform P_λ for the hyperbolic spaces $B(\mathbb{F}^n)$ over $\mathbb{F} = \mathbb{R}, \mathbb{C}$ or the quaternions $\mathbb{H}$. Namely, if Δ is the Laplace-Beltrami operator of $B(\mathbb{F}^n)$ and sF a $\mathbb{C}$-valued function on $B(\mathbb{F}^n)$ satisfying $\Delta F = -(\lambda^2 + \sigma^2)F; \lambda \in \mathbb{R}^*$ then we establish: i) F is the Poisson transform of some $f \in L^2(\partial B(\mathbb{F}^n))$ (ie $P_\lambda f = F$) if and only if it satisfies the growth condition:

$$\sup_{t>0} \frac{1}{t} \int_{B(0,t)} |F(x)|^2 d\mu(x) < +\infty,$$

where $B(0,t)$ is the ball of radius t centered at 0 and $d\mu$ the invariant measure on $B(\mathbb{F}^n)$. ii) F is the Poisson transform of some $f \in L^p(\partial B(\mathbb{F}^n))$, $p \geq 2$; if and only if it satisfies the following Hardy-type growth condition:

$$\sup_{0 \leq r < 1} (1-r^2)^{-\frac{\sigma}{2}} \left(\int_{\partial B(\mathbb{F}^n)} |F(r\theta)|^p d\theta \right)^{\frac{1}{p}} < +\infty.$$

Keywords: Hyperbolic spaces, Poisson transform, Calderon Zygumund estimates, Jacobi functions.