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# Fuss Relations and Primitive Factorization for the Bicentric 21-gon

We study the Cayley Poncelet closing condition for the bicentric 21-gon and its factorization in the dimensionless variables $p = (R + d)/r$ and $q = (R - d)/r$. Two explicit tangent constructions, with rotation numbers $k = 1$ and $k = 2$, give representative 21-step closing polynomials in $Q[p, q[$. Each such polynomial has bidegree $(55, 55)$ and contains factors corresponding to closing periods 3 and 7. After these lower-period factors are removed, the two representative Fuss relations $F_{21}^{(1)}$ and $F_{21}^{(2)}$ are the primitive period-21 factors; each has bidegree $(48, 48)$ and 1225 non-zero monomials. The independently derived $k = 2$ full closing polynomial is shown to equal the polynomial obtained from the $k = 1$ full closing polynomial by the substitution $(p, q) \mapsto (-p, -q)$, and the same substitution relates the two primitive factors. We also verify the exact factorization of the Cayley determinant for $n = 21$. Rotation numbers with $\gcd(21, k) > 1$ reduce to the period-3 or period-7 closing conditions, while for the remaining coprime rotation numbers only the representatives $k = 1$ and $k = 2$ are derived symbolically in this paper.

**Keywords**: Bicentric polygon, Fuss relation, Poncelet polygon, Cayley determinant, primitive factorization, rotation number.

**MSC**: 51M04; 51M15, 51N20, 14H52, 14Q20.