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A Volume Comparison for Parallel and Orthogonal Projections in a Simplex

Let $SA_1 \cdots A_n$ be a nondegenerate simplex in Euclidean n-space, where $n \geq 2$, and let P be any point. For each i, let h_i be the facet hyperplane opposite A_i. Denote by X_i and U_i the projections of P onto h_i, respectively parallel to SA_i and orthogonal to h_i. We prove that

$$\mathrm{Vol}_n(PX_1 \cdots X_n) \geq \mathrm{Vol}_n(PU_1 \cdots U_n).$$

If P lies on one of the hyperplanes h_i, both volumes are zero. Otherwise, equality holds if and only if the edges through S are pairwise orthogonal. The proof uses a reciprocal basis and Hadamard's determinant inequality.

Keywords: Simplex, pedal simplex, volume comparison, parallel projection, orthogonal projection, reciprocal basis, Hadamard inequality.

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