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A Family of Eight-Point Conics Associated with the Cyclic Quadrilateral

We consider the following configuration. Let $ABCD$ be a cyclic quadrilateral with circumcenter O, and for each vertex X, let H_X be the orthocenter of the triangle formed by the other three. Then A, B, C, D, H_A, H_B, H_C, H_D all lie on a single conic. In this paper we study a certain generalization of this fact as follows. For an arbitrary point P_D on the Euler line of $\triangle ABC$, we define corresponding points P_A, P_B, P_C on the respective Euler lines such that the ratio $P_X H_X : P_X O$ is constant for all X. We show that the four vertices A, B, C, D and the four isogonal conjugates Q_A, Q_B, Q_C, Q_D of the points P_X all lie on a single conic. This result is given distinct treatments, synthetic, projective, and algebraic. Furthermore, we situate the points P_X within the list of triangle centers.

Keywords: Euler line, isogonal conjugate, conic, cyclic quadrilateral, triangle center, orthocenter, Shinagawa coefficients.

MSC: 51M04; 51M05, 51M15.