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More Variations on Fermat Analogue of the Steiner-Lehmus Theorem

The celebrated Steiner-Lehmus theorem states that if the internal bisectors of two angles of a triangle are equal, then the corresponding sides have equal lengths. In this paper, we consider the triangle ABC whose all angles are less than 120°, F is its Fermat point, and per(ABC), [ABC] stand for its perimeter and area, respectively. In Theorem 1, we prove the Fermat analogue of Steiner-Lehmus Theorem that states that if the cevians from B and C through the Fermat point F meet AC and AB at B' and C' respectively, then $BB' = CC'$ is equivalent to $AB = AC$. More stronger forms are also proved such as $AB > AC$ is equivalent to each of $BB' > CC'$ and per($C'BC$ > per($B'CB$). More variations on Fermat analogue of Steiner-Lehmus Theorem are proved in Theorems 3 and 4. In Theorem 3, the cevians through F from B and C meet the external angle bisectors of C and B at D and E respectively, and it is proved that, for example, $AB = AC$ is equivalent to each of $CE = BD$, per($EC'B$) = per($DB'C$), and [$EC'B$] = [$DB'C$] and more stronger forms are also proved such as $AB > AC$ is equivalent to each of $CE > BD$, per($EC'B$) > per($DB'C$), and [$EC'B$] > [$DB'C$]. In Theorem 4, we prove that if the angle A of the triangle ABC is not equal to 60° and the circumcevians BK and CL of the Fermat point F, that meet the circumcircle of $\triangle ABC$ at K and L, are equal, then the triangle $\triangle ABC$ is isosceles with $AB = AC$.

Keywords: Steiner-Lehmus Theorem, Fermat point, cevian, circumcevian.

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