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H. Katsuura
San Jose State University, San Jose, U.S.A.
hidefumi.katsuura@sjsu.edu

Central and Twin Tetrahedra

Given a tetrahedron T, the tetrahedron T' constructed by connecting the four centroids of its faces is called the *central tetrahedron* of T. A tetrahedron T can be inscribed in a parallelepiped W so that the edges of T are the diagonals of the faces of W. By drawing the remaining six diagonals on the faces of the parallelepiped W, we obtain a new tetrahedron $T^\star$, and call it the *twin tetrahedron* of T. Let $S^\star$ and $S^{\star\prime}$ be the circumcenters of $T^\star$ and $T^{\star\prime}$, respectively. We will prove that all tetrahedra T, T', $T^\star$, and $T^{\star\prime}$ have the centroid in common, say P, and the five points S, $S^{\star\prime}$, P, S', and $S^\star$ are collinear in this order such that $\overrightarrow{S'S^\star} = 2\overrightarrow{PS'}$, $\overrightarrow{SP} = 3\overrightarrow{PS'}$, $\overrightarrow{SS'} = 2\overrightarrow{S'S^\star}$, and $\overrightarrow{SS^\star} = 3\overrightarrow{S'S^\star}$. Moreover, we prove that (1) T' and $T^{\star\prime}$ are twins, and (2) if the tetrahedron T is orthocentric, then T, T', $T^\star$, $T^{\star\prime}$ are orthocentric with orthocenters $S^\star$, $S^{\star\prime}$, S, and S', respectively.

Keywords: Central tetrahedron, twin tetrahedron, centroid, circumcenter, orthocentric tetrahedron, orthocenter.

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