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Packings with Geodesic and Translation Balls and Their Visualizations in $\widetilde{\mathbf{SL}_2\mathbf{R}}$ Space

Remembering our friendly cooperation between the Geometry Departments of the Technical Universities of Budapest and Vienna (also under different names) a nice topic comes into my memory: the Gum fibre model , a model made of fibres and two disks of the hyperbolic base plane as it is well-known as the surface of a cooling tower of a power plant.
One point of view is the so-called kinematic geometry by the Vienna colleagues, e.g., as in a paper by H. Stachel [*Flexible octahedra in hyperbolic space*, in: *Non-Euclidean Geometries*, A. Prekopa and E. Molnar (eds.), Janos Bolyai Memorial Volume 581, Springer, Boston (2006) 209–225], but also in a very general context. The other point is the so-called $\mathbf{H}^2 \times \mathbf{R}$ geometry and $\widetilde{\mathbf{SL}_2\mathbf{R}}$ geometry where – roughly – two hyperbolic planes as circle discs are connected with gum fibres, first: in a simple way, second: in a twisted way.
This second homogeneous (Thurston) geometry will be our topic (initiated by some Budapest colleagues, and discussed also in international cooperations). We use for the computation and visualization of $\widetilde{\mathbf{SL}_2\mathbf{R}}$ its projective model, as in some previous papers. We found a seemingly extremal geodesic ball packing for the $\widetilde{\mathbf{SL}_2\mathbf{R}}$ group $\mathbf{pq}_k\mathbf{o}_\ell$ ($p = 9$, $q = 3$, $k = 1$, $o = 2$, $\ell = 1$) with density ≈ 0.787758. A much better translation ball packing was found for the group $\mathbf{pq}_k\mathbf{o}_\ell$ ($p = 11$, $q = 3$, $k = 1$, $o = 2$, $\ell = 1$) with density ≈ 0.845306.

Keywords: Thurston geometries, SL2R geometry, density of ball packing under space group, regular prism tiling, volume in SL2R.

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