H. Stachel

Institute of Discrete Mathematics and Geometry, Vienna University of Technology, Vienna, Austria
stachel@dmg.tuwien.ac.at

Recalling Thread Constructions of Quadrics

The role of quadrics in Euclidean 3-space is similar to that of conics. Therefore, it is natural to ask for thread constructions of quadrics, as spatial analogues of the gardener's construction or Graves' construction of ellipses. The first solution given in 1882 by O. Staude is based on an ellipse e and its focal hyperbola h. A thread of given length, fixed with one end at a focal point of h, is passed behind the nearest branch of h and in front of e and finally attached to the vertex of the second branch of h. If the thread is stretched at a point P between, then P traces a patch of an ellipsoid $\mathcal{E}$ confocal with e and h. Later, Staude presented a second type of thread constructions where e and h are replaced by an ellipsoid $\mathcal{E}_0$ and a confocal hyperboloid H_0. Here, the thread follows at its ends the two branches of the line of curvature $\mathcal{E}_0 \cap \mathrm{H}_0$. We provide a synthetic approach to these constructions and discuss the case of paraboloids.

Keywords: Quadric, thread construction, focal conics, confocal quadrics.

MSC: 51M04; 53A05, 53A17