Journal for Geometry and Graphics 23 (2019) 245–258

J. Wittenburg
Institute for Technical Mechanics, Karlsruhe Institute of Technology, Kaiserstr. 12, 76128 Karlsruhe, Germany
jens.wittenburg@kit.edu

Foldable and Self-Intersecting Polyhedral Cylinders Based on Triangles

An infinitely long strip of paper is divided by a zigzagging line into congruent triangles with side lengths 1, a and b. On both rims of the strip the vertices V_k of the triangles are labeled from $-\infty$ to $+\infty$ with a shift n such that $(V_0V_1V_n)$ is a representative triangle. Along the sides of the triangles folds with alternating fold angles are made. Under certain conditions on a, b and n and with appropriately chosen fold angles it is possible to bring every vertex V_k on the upper rim in coincidence with the vertex V_k of equal name on the lower rim. The resulting body is a polyhedral cylinder (PC). The vertices are distributed at equal intervals along a helix on the surface of a circular cylinder. For given lengths a and b up to $(n-2)$ PCs can be formed. There are foldable PCs and self-intersecting PCs. In the case $n = 4$ self-intersecting PCs consist of a core body with congruent nonconvex pentagonal faces and of an infinite number of congruent tetrahedra, each tetrahedron in edge-to-edge contact with the core body along three edges.

Keywords: Polyhedral cylinder, core body, foldability, flexible polyhedra, periodic framework.

MSC: 52C25; 53A17, 51M20