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I. Minevich

Dept. of Mathematics, Boston College, 140 Commonwealth Avenue, Chestnut Hill, MA 02467-3806, U.S.A.
igor.minevich@bc.edu

P. Morton

Dept. of Mathematical Sciences, Indiana & Purdue University, 402 N. Blackford Street, Indianapolis, IN 46202, U.S.A.
pmorton@math.iupui.edu

Vertex Positions of the Generalized Orthocenter and a Related Elliptic Curve

We study triangles ABC and points P for which the generalized orthocenter H corresponding to P coincides with a vertex. The set of all such points P is a union of three ellipses minus six points. If T_P is the affine map taking ABC to the cevian triangle of P , P' is the isotomic conjugate of P , and K is the complement map for ABC , we also study the affine map $M_P = T_P \circ K^{-1} \circ T_{P'}$ taking the circumconic of ABC for P to the inconic of ABC for P . We show that the locus of points P for which this map is a translation is an elliptic curve minus six points, and show how this locus can be synthetically constructed using the geometry of the triangle.

Keywords: Generalized orthocenter, circumconic, inconic, affine maps, elliptic curve.

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