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Counting Escher's m×m Ribbon Patterns

Using a construction scheme originally devised by M.C. Escher, one can generate doubly-periodic patterns of the xy-plane with the operations of rotation, reflection and translation acting on an asymmetric square motif. Rotating and/or reflecting the original motif yields eight distinct aspects. By selecting m^2 (not necessarily distinct) motif aspects and arranging them in an $m \times m$ Escher tile, one can then tile the xy-plane by translating the Escher tile by integer multiples of m in the x and/or y direction to create wallpaper patterns.
Two wallpaper patterns are considered equivalent if there is some isometry between the two. Previously, the general formula was given by the second author [Proc. 32nd Southeastern Conf. on Combinatorics, Graph Theory and Computing, Baton Rouge, vol. 153 (2001) 77–96] for the number of inequivalent patterns generated by $m \times m$ Escher tiles composed of the four rotated aspects of a single asymmetric motif by applying Burnside's Lemma. Here we extend that formula to include the four additional reflected aspects when composing $m \times m$ Escher tiles with which to tile the plane.

Keywords: Motif, wallpaper pattern, Escher tile, symmetry, group action, geometric structure.

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