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**On Abstract Convexity with Respect to a Certain Quadratic Coupling
Function: Subsets**

We investigate abstract convexity generated by the coupling function

$$\varphi(x, A) = |A[x]^2|, \quad x \in \mathbb{R}^n, \quad A \in \mathcal{L}_{sym}^2(\mathbb{R}^n, \mathbb{R}).$$

We study the φ -convexity of four classes of sets: subsets of $\mathbb{R}^n$, subsets of $\mathcal{L}$, epigraphic subsets of $\mathbb{R}^n \times \mathbb{R}$ and epigraphic subsets of $\mathcal{L} \times \mathbb{R}$. We show that all families consist of closed, star-shaped sets and subsets of $\mathcal{L}$ and $\mathcal{L} \times \mathbb{R}$ are convex. For epigraphic subsets of $X \times \mathbb{R}$, we derive a condition partially characterizing them in analogy with classical convexity, where line segments are replaced by parabolas. In the one-dimensional case we obtain a complete characterization.

Keywords: Abstract convexity, coupling function, bilinear maps, quadratic functions.

MSC: 52A01, 52A30, 15A63.