Journal of Convex Analysis 34 (2027) 183–194

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A Cautionary Tale – Every Set is an Interval Set and an Equiset

Given a metric space (X, d) with distinct points $x, y \in X$, we consider the interval set $I_d(x, y) = \{z \in X : d(x, z) + d(z, y) = d(x, y)\}$ and the equiset $E_d(x, y) = \{z \in X : d(x, z) = d(y, z)\}$. The topology and geometry of these regions have been studied, in particular when the metric arises from a norm. In this note, we consider the question of which sets arise as interval sets and equisets in general metrizable spaces. We show that, subject to some obvious restrictions, every closed set in every metrizable space occurs as an interval set or equiset, with respect to some compatible metric. We also consider the case of geodesic spaces, and show that any arc or closed union of geodesics between two points of a complete locally compact geodesic space arises as the interval set with respect to some compatible geodesic metric.

Keywords: Metric spaces, equivalent metrics, geodesic spaces, Menger intervals, interval sets, midsets, equisets, comparative nearness regions, Menger betweenness, convex metrics.

MSC: 52A01, 54E35, 51F99, 51K06.