Journal of Convex Analysis 34 (2027) 001–018

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Mosco-Convergence of Convex Sets and Unilateral Problems for Differential Operators with Lower Order Terms Having Natural Growth

We study the stability of solutions to a class of variational inequalities posed on obstacle-type convex sets, under Mosco-convergence. Specifically, we consider problems of the form

$$\begin{cases} u \in W_0^{1,p}(\Omega) \cap L^\infty(\Omega), & u \geq \psi \quad \text{in } \Omega, \\ \langle A(u), v-u \rangle + \displaystyle\int_\Omega H(x,u,Du)(v-u) \geq 0, & \\ \forall\, v \in W_0^{1,p}(\Omega) \cap L^\infty(\Omega), & v \geq \psi \quad \text{in } \Omega. \end{cases}$$

Here, A is a Leray Lions type operator, mapping $W_0^{1,p}(\Omega)$ into its dual $W^{-1,p'}(\Omega)$, while $H(x,u,Du)$ grows like $|Du|^p$. The obstacle ψ is a function in $W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$. Our main result establishes that the solutions are stable under Mosco-convergence of the constraint sets. This extends classical stability results to natural growth problems.

Keywords: Mosco-convergence, variational inequalities, obstacle-type convex sets, Leray-Lions operators, operators with natural growth terms, stability of solutions.

MSC: 35J87, 47J20, 49J40.