Journal of Convex Analysis 33 (2026) 737–764

A. Braides
(1) Department of Mathematics, University Tor Vergata, Rome, Italy
(2) SISSA, Trieste, Italy
abraides@sissa.it

S. Scalabrino
SISSA, Trieste, Italy
sscalabr@sissa.it

C. Trifone
SISSA, Trieste, Italy
ctrifone@sissa.it

Homogenization of Non-Local Energies on Disconnected Sets

We consider the problem of the homogenization of non-local quadratic energies defined on δ-periodic disconnected sets defined by a double integral, depending on a kernel concentrated at scale ε. For kernels with unbounded support we show that we may have three regimes: (i) $\varepsilon \ll \delta$, for which the Γ-limit even in the strong topology of L^2 is 0; (ii) $\frac{\varepsilon}{\delta} \to \kappa$, in which the energies are coercive with respect to a convergence of interpolated functions, and the limit is governed by a non-local homogenization formula parameterized by κ; (iii) $\delta \ll \varepsilon$, for which the Γ-limit is computed with respect to a coarse-grained convergence and exhibits a separation-of-scales effect; namely, it is the same as the one obtained by formally first letting $\delta \to 0$ (which turns out to be a pointwise weak limit, thanks to an iterated use of Jensen's inequality), and then, noting that the outcome is a nonlocal energy studied by Bourgain, Brezis and Mironescu, letting $\varepsilon \to 0$. A slightly more complex description is necessary for case (ii) if the kernel is compactly supported.

Keywords: Homogenization, Gamma-convergence, perforated domains, separation of scales, non-local functionals.

MSC: 35B27, 74Q05, 49J45, 26A33, 74A70.