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The Ever Large Subspace $C_p(Y|X)$: Distinguished, Montel, Covered Nicely?

$C_p(Y|X)$ denotes the real-valued continuous functions on $Y \subseteq X$ having continuous extensions to a Tychonoff space X, with pointwise topology inherited from $C_p(Y)$. We recently proved $C_p(Y)$ *is distinguished* $\Leftrightarrow$ *it is a large subspace of* $\mathbb{R}^Y$. We prove $C_p(Y|X)$ *is always a large subspace of* $C_p(Y)$. Thus $C_p(Y|X)$ *is always quasibarrelled; always has a feral strong dual; has a quasibarrelled countable enlargement* $\Leftrightarrow$ Y *is infinite; is distinguished* $\Leftrightarrow$ $C_p(Y)$ *is distinguished; is a Montel space* $\Leftrightarrow$ Y *is discrete and* C*-embedded in* X. 'Nice' countable covers for $C_p(Y|X)$ yield potent summary theorems that solve open problems, characterize P-spaces anew, and complete the list of Velichko variations. For example, Summary III: *Assume* Y *is dense in* X. Y *is a* P*-space, or* X *is pseudocompact, or both* $\Leftrightarrow$ $C_p(Y|X)$ *is countably covered by sets that are, respectively, relatively sequentially complete in* $C_p(Y)$*, or bounded, or both.* Putting $Y = X$, one quickly comprehends Velichko variations *à la* Arkhangel'skiĭ.

Keywords: Large subspace, distinguished, Montel, quasibarrelled, $C_p(X)$, C-embedded.

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