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The Lions Concentration-Compactness Principle for the Dirichlet Problem for Partial Differential Equations with Variable Exponent Laplace Operator

We develop the P. Lions concentration-compactness principle for a sequence of Radon measures on R^n, the P. Lions principle is extended to variable exponent Lebesgue spaces $L^{p(\cdot)}(\Omega)$, $\Omega \subseteq R^n$, $n \geq 3$. Employing this $L^{p(\cdot)}$-extension of the concentration-compactness principle, we establish almost exact conditions under which the Dirichlet problem $u|_{\partial\Omega} = 0$ for variable exponent Laplace equation

$$-div\left(|\nabla u|^{p(x)-2}\nabla u\right) + \lambda\,|u|^{p(x)-2}\,u = a\,(x)\,|u|^{s(x)-2}\,u + f\,(x,\ u)$$

has a weak solution in variable exponent Sobolev space $W_1^{p(\cdot)}(\Omega)$, with critically grown coefficients.

Keywords: Concentration-compactness, variable exponent Lebesgue spaces, variable exponent Sobolev space, mountain pass theorem, concentration-compactness principle, Radon measure, Levy distribution.

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