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Spectral Study about Biharmonic Operator with Landesman-Lazer Condition

We study the existence of weak solutions for the following class of problems

$$\begin{cases} \alpha\Delta^2 u + \beta\Delta u = \mu u + \gamma h(x,u) & \text{in } \Omega, \\ B(u) = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $N \geq 1$, $\alpha \geq 0$, $-\infty < \beta < \alpha\lambda_1$, λ_1 is the first eigenvalue of $(-\Delta, H_0^1(\Omega))$, $\mu \in (0,\bar{\mu})$, $\bar{\mu} < \mu_2$, μ_2 is the second eigenvalue of the problem

$$(\alpha\Delta^2 u + \beta\Delta u, H_0^1(\Omega) \cap H^2(\Omega)),$$

$\gamma \neq 0$ is a real parameter and $h : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function verifying some conditions, the boundary condition $B(u) = 0$ on $\partial\Omega$ means that $u = \Delta u = 0$ on $\partial\Omega$ when $\alpha > 0$ and $u = 0$ on $\partial\Omega$ when $\alpha = 0$. In this article, we revisit the arguments of Landesman-Lazer, both in the global and local aspects.

Keywords: Biharmonic operator, Landesman-Lazer condition.

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