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Convex Interval Hull of Finite Sets in Real Linear Spaces: Extreme Points and Unbounded Images

Let S be a finite set in a real linear space and let $\mathcal{J}_S$ be a family consisting of $|S|$ intervals in $\mathbb{R}$. In this paper we deal with a convex operator $\operatorname{co}(S, \mathcal{J}_S)$ called the convex interval hull. This operator generalizes the familiar concepts of the convex hull, $\operatorname{conv}(S)$, and the affine hull, $\operatorname{aff}(S)$, of S. The set $\operatorname{co}(S, \mathcal{J}_S)$ is a convex subset of the linear space and can be either bounded or unbounded, depending on the families $\mathcal{J}_S$. In this paper we apply $\operatorname{co}(S, \mathcal{J}_S)$ to obtain unbounded images of a finite set S. As special images of $\operatorname{co}(S, \mathcal{J}_S)$ for finite S we obtain such unbounded objects as: hyperplanes, cylinders, cones, penumbras and wedges. We also apply $\operatorname{co}(S, \mathcal{J}_S)$ to study some properties of extreme points. In relation to $\operatorname{co}(S, \mathcal{J}_S)$ we introduce the so-called extreme interval operator $\operatorname{Eco}(S)$ and prove some analogues of the celebrated Minkowski-Krein-Milman's theorem.

Keywords: Convex set, cone, convex interval hull, extreme point, extreme interval operator, Minkowski-Krein-Milman property.

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