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Asymptotics of the p-Capacity in the Critical Regime

We are interested in the asymptotics of the p-capacity between the origin and the set nB, where B is the boundary of the unit ball of the lattice $\mathbb{Z}^d$. The p-capacity is defined as the minimum of the Dirichlet energy associated with a discrete version of the p-Laplacian. This variational problem has arisen in particular in the study of large deviations for first passage percolation. For $p < d$, the p-capacity converges to some positive constant, while for $p > d$ the capacity vanishes polynomially fast. The present paper deals with the case $p = d$, for which we prove that the p-capacity vanishes as $c_d(\log n)^{-d+1}$ with an explicit constant c_d. Our proof relies on Thomson's principle for the p-capacity.

Keywords: p-Capacity, variational problem, first passage percolation.

MSC: 31C45; 31C20, 94C15, 60K35.