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## **A Uniqueness Result for a Translation Invariant Problem in the Calculus of Variations**

We present a uniqueness result of uniformly continuous solutions for a general minimization problem in the Calculus of Variations. We minimize the functional  $\mathcal{I}_\lambda(u) := \int_\Omega \varphi(\nabla u) + \lambda u$  with  $\varphi$  a convex but not necessarily strictly convex function,  $\Omega$  an open set of  $\mathbb{R}^N$  with  $N \in \mathbb{N}$  and  $\lambda \in \mathbb{R}$ . The proof is based on the two following main points: the functional  $\mathcal{I}_\lambda$  is invariant under translations and we assume that the function  $\varphi$  is not affine on any non-empty open set. This provides a shorter proof and/or an extension for some already known uniqueness results for functionals of the type  $\mathcal{I}_\lambda$  that are presented in the article.

**Keywords:** Calculus of variations, translation invariance, non strictly-convex function, uniqueness.

**MSC:** 35A02, 49N99.