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**Cone-Constrained Singular Value Problems**

The singular values of a matrix  $A$  of size  $m \times n$  can be seen as the critical values of the bilinear form  $\langle u, Av \rangle$  with  $u$  and  $v$  ranging over the unit spheres of  $\mathbb{R}^m$  and  $\mathbb{R}^n$ , respectively. If  $u$  and  $v$  are further restricted by closed convex cones  $P$  and  $Q$ , respectively, then the criticality conditions are:  $P \ni u \perp (Av - \sigma u) \in P^*$ ,  $Q \ni v \perp (A^\top u - \sigma v) \in Q^*$ . This is a coupled system of complementarity problems involving a pair of cones and their dual cones. The parameter  $\sigma$  is called a singular value of  $A$  relative to  $(P, Q)$ . The purpose of our work is to study this new concept of singular value. The analysis of such a coupled system is motivated by a number of applications. By way of illustration, we consider a nonnegative Principal Component Analysis problem.

**Keywords:** Convex cone, cone-constrained singular value, cone-constrained eigenvalue, complementarity problem, principal component analysis.

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