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Journal of Convex Analysis 30 (2023) 1053–1072

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Lebesgue Infinite Sums of Convex Functions: Subdifferential Calculus

We present a subdifferential analysis for a general concept of infinite sum $f := \sum_{i \in I} f_i$ of arbitrary collections of convex functions f_i , called Lebesgue infinite sum. Since this problem cannot be addressed, at least directly, through classical arguments from the theory of normal convex integrands, we perform a reduction analysis showing that the ε -subdifferential of f reduces to that of countable/finite subsums via appropriate lower limit and closure processes. Then, the usual calculus rules of (countable) integral functions give rise to characterizations of the ε -subdifferential of f , which are written exclusively by means of ε -subdifferentials of the data f_i . The resulting characterizations do not assume any qualification or boundedness condition.

Keywords: Lebesgue infinite sum, convex functions, subdifferential calculus.

MSC: 26B05, 26J25, 49H05.