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Orthant-Strictly Monotonic Norms, Generalized Top- k and k -Support Norms and the ℓ_0 Pseudonorm

The so-called ℓ_0 pseudonorm on the Euclidean space \mathbb{R}^d counts the number of nonzero components of a vector. We say that a sequence of norms is strictly increasingly graded (with respect to the ℓ_0 pseudonorm) if it is nondecreasing and that the sequence of norms of a vector x becomes stationary exactly at the index $\ell_0(x)$. In this paper, with any (source) norm, we associate sequences of generalized top- k and k -support norms, and we also introduce the new class of orthant-strictly monotonic norms (that encompasses the ℓ_p norms, but for the extreme ones). Then, we show that an orthant-strictly monotonic source norm generates a sequence of generalized top- k norms which is strictly increasingly graded. With this, we provide a systematic way to generate sequences of norms with which the level sets of the ℓ_0 pseudonorm are expressed by means of the difference of two norms. Our results rely on the study of orthant-strictly monotonic norms.

Keywords: ℓ_0 pseudonorm, orthant-strictly monotonic norm, generalized top- k norm, generalized k -support norm, strictly graded sequence of norms.

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