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Embedding of Topological Posets in Hyperspaces

We study the problem of topologically order-embedding a given topological poset $(X, \preceq)$ in the space of all closed subsets of X which is topologized by the Fell topology and ordered by set inclusion. We show that this can be achieved whenever $(X, \preceq)$ is a topological semilattice (resp. lattice) or a topological po-group, and X is locally compact and order-connected (resp. connected). We give limiting examples to show that these results are tight, and provide several applications of them. In particular, a locally compact version of the Urysohn-Carruth metrization theorem is obtained, a new fixed point theorem of Tarski-Kantorovich type is proved, and it is found that every locally compact and connected Hausdorff topological lattice is a completely regular ordered space.

Keywords: Topological poset, hyperspace, Fell topology, topological semilattice, topological po-group, topological order-embedding, radially convex metric, complete semilattice homomorphism.

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