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Preservation or Not of the Maximally Monotone Property by Graph-Convergence

In a general real Hilbert space $\mathcal{H}$, given a sequence $(A_n)_{n\in\mathbb{N}}$ of maximally monotone operators $A_n : \mathcal{H} \rightrightarrows \mathcal{H}$, which graphically converges to an operator A whose domain is nonempty, we analyze if the limit operator A is still maximally monotone. This question is justified by the fact that, as we show on an example in infinite dimension, the graph limit in the sense of Painlevé-Kuratowski of a sequence of maximally monotone operators may not be maximally monotone. Indeed, the answer depends on the type of graph convergence which is considered. In the case of the Painlevé-Kuratowski convergence, we give a positive answer under a local compactness assumption on the graphs of the operators A_n. Under this assumption, the sequence $(A_n)_{n\in\mathbb{N}}$ turns out to be convergent for the bounded Hausdorff topology. Inspired by this result, we show that, more generally, when the sequence $(A_n)_{n\in\mathbb{N}}$ of maximally monotone operators converges for the bounded Hausdorff topology to an operator whose domain is nonempty, then the limit is still maximally monotone. The answer to these questions plays a crucial role in the sensitivity analysis of monotone variational inclusions, and makes it possible to understand these questions in a unified way thanks to the concept of proto-differentiability. It also leads to revisit several notions which are based on the convergence of sequences of maximally monotone operators, in particular the notion of variational sum of maximally monotone operators.

Keywords: Maximally monotone operator, graph convergence, bounded Hausdorff convergence, proto-differentiability, sensitivity analysis, variational inclusion, variational sum.

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