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On Concentration Behavior and Multiplicity of Solutions for a System in $\mathbb{R}^N$

We describe a result on the asymptotic behavior of the solutions of a system with two elliptic equations in $\mathbb{R}^N$ involving a small parameter. More precisely, we study the system

$$\begin{cases} -\varepsilon^2 \mathrm{div}(a(x)\nabla u) + u = Q_u(u,v) + \dfrac{\gamma}{2^*} K_u(u,v) \ \text{ in } \mathbb{R}^N, \\ -\varepsilon^2 \Delta v + b(x)v = Q_v(u,v) + \dfrac{\gamma}{2^*} K_v(u,v) \ \text{ in } \mathbb{R}^N, \\ u, v \in H^1(\mathbb{R}^N),\ u(x),\ v(x) > 0 \ \text{ for each } x \in \mathbb{R}^N, \end{cases}$$

where $2^* = 2N/(N-2)$, $N \geq 3$, $\varepsilon > 0$, a and b are positive continuous potentials, and Q and K are homogeneous functions with K having critical growth. We use the penalization method for system introduced by C. O. Alves [*Local mountain pass for a class of elliptic system*, J. Math. Analysis Appl. 335 (2007) 135–150] in order to find a family of solutions $(u_\varepsilon, v_\varepsilon)$ in $H^1(\mathbb{R}^N) \times H^1(\mathbb{R}^N)$ such that, if $\Pi_{\varepsilon,a}$ and $\Pi_{\varepsilon,b}$ are maximum points of u_ε and v_ε respectively, then

$$\lim_{\varepsilon \to 0^+} a(\Pi_{\varepsilon,a}) = \inf_{x \in \mathbb{R}^N} a(x) \quad \text{and} \quad \lim_{\varepsilon \to 0^+} b(\Pi_{\varepsilon,b}) = \inf_{x \in \mathbb{R}^N} b(x).$$

Moreover, we relate the number of solutions with the topology of the set where the potentials a and b attain their minima. We consider the subcritical case $\gamma = 0$ and the critical case $\gamma = 1$.

Keywords: Elliptic systems, Schroedinger equation, Ljusternik-Schnirelmann theory, positive solutions.

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