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Existence of Positive Solutions for a Critical Nonlocal Elliptic System

We establish the existence of positive solution to the critical nonlocal elliptic system

$$(S)\qquad \begin{cases} (-\Delta)_p^s u + a(x)|u|^{p-2}u + c(x)|v|^{p-2}v = \frac{1}{p_s^*}K_u(u,v) \ \text{ in } \ \mathbb{R}^N, \\ (-\Delta)_p^s v + c(x)|u|^{p-2}u + b(x)|v|^{p-2}v = \frac{1}{p_s^*}K_v(u,v) \ \text{ in } \ \mathbb{R}^N, \\ u, v > 0 \text{ in } \mathbb{R}^N, \ u, v \in D^{s,p}(\mathbb{R}^N), \ N > ps, \ s \in (0,1). \end{cases}$$

Here $(-\Delta)_p^s$ denotes the fractional p-Laplacian, a, b and c are suitable functions and K is a p_s^*-homogeneous function, $p_s^* = (pN)/(N-ps)$, $N > ps$. One of the main tools is to apply the global compactness result for the associated energy functional similar to that due to M. Struwe [*A global compactness result for elliptic boundary value problems involving limiting nonliarities*, Math. Zeitschrift 187/4 (1984) 511–517] combined with some information on a limit system of (S) with $a = b = c = 0$, the concentration compactness due to P. L. Lions [*The concentration-compactness principle in the calculus of variations. I: The limit case*, Rev. Mat. Iberoamericana 1/1 (1985) 145–201] and the Brouwer degree theory.

Keywords: Variational critical system, fractional equations, global compactness result, Brouwer degree theory.

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