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On Property- (P_1) in Banach Spaces

We discuss a set-valued generalization of strong proximality in Banach spaces, introduced by J. Mach [*Continuity properties of Chebyshev centers*, J. Approx. Theory 29/3 (1980) 223–230] as property- (P_1) . For a Banach space X , a closed convex subset V of X and a subclass \mathcal{F} of the closed bounded subsets of X , this property, defined for the triplet (X, V, \mathcal{F}) , describes simultaneous strong proximality of V at each of the sets in \mathcal{F} . We establish that if the closed unit ball of a closed subspace of a Banach space X possesses property- (P_1) for each of the classes of closed bounded, compact and finite subsets of X , then so does the subspace. It is also proved that the closed unit ball of an M -ideal in an L_1 -predual space satisfies property- (P_1) for the compact subsets of the space. For a Choquet simplex K , we provide a sufficient condition for the closed unit ball of a finite co-dimensional closed subspace of $A(K)$ to satisfy property- (P_1) for the compact subsets of $A(K)$. This condition also helps to establish the equivalence of strong proximality of the closed unit ball of a finite co-dimensional subspace of $A(K)$ and property- (P_1) of the closed unit ball of the subspace for the compact subsets of $A(K)$. Further, for a compact Hausdorff space S , a characterization is provided for a strongly proximal finite co-dimensional closed subspace of $C(S)$ in terms of property- (P_1) of the subspace and that of its closed unit ball for the compact subsets of $C(S)$. We generalize this characterization for a strongly proximal finite co-dimensional closed subspace of an L_1 -predual space. As a consequence, we prove that such a subspace is a finite intersection of hyperplanes such that the closed unit ball of each of these hyperplanes satisfy property- (P_1) for the compact subsets of the L_1 -predual space and vice versa. We conclude this article by providing an example of a closed subspace of a non-reflexive Banach space which satisfies $1\frac{1}{2}$ -ball property and does not admit restricted Chebyshev center for a closed bounded subset of the Banach space.

Keywords: Property- (P_1) , strong proximality, restricted Chebyshev center, L_1 -predual, M -ideal, $1\frac{1}{2}$ -ball property.

MSC: 41A65, 41A50; 52A07, 46E15.