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Unique Minimizers and the Representation of Convex Envelopes in Locally Convex Vector Spaces

It is well known that a strictly convex minimand admits at most one minimizer. We prove a partial converse: Let X be a locally convex Hausdorff space and $f\colon X \to (-\infty, \infty]$ a function with compact sublevel sets and exhibiting some mildly superlinear growth. Then each tilted minimization problem

$$\min_{x \in X} f(x) - \langle x', x \rangle_X$$

admits at most one minimizer as x' ranges over dom (∂f^*) if and only if the biconjugate f^{**} is essentially strictly convex and agrees with f at all points where f^{**} is subdifferentiable. We prove this via a representation formula for f^{**} that might be of independent interest.

Keywords: Locally convex Hausdorff space, (essentially) strictly convex function, biconjugate, convex envelope, convex hull, subdifferential, uniqueness.

MSC: 46G05, 52A07, 46N10, 49N15.