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Functions on a Convex Set which are both ω-Semiconvex and ω-Semiconcave

Let $G \subset \mathbb{R}^n$ be an open convex set which is either bounded or contains a translation of a convex cone with nonempty interior. It is known that, for every modulus ω, every function on G which is both semiconvex and semiconcave with modulus ω is (globally) $C^{1,\omega}$-smooth. We show that this result is optimal in the sense that the assumption on G cannot be relaxed. We also present direct short proofs of the above mentioned result and of some its quantitative versions. Our results have immediate consequences concerning (i) a first-order quantitative converse Taylor theorem and (ii) the problem whether $f \in C^{1,\omega}(G)$ whenever f is continuous and smooth in a corresponding sense on all lines. We hope that these consequences are of an independent interest.

Keywords: ω-semiconvex functions, ω-semiconcave functions, $C^{1,\omega}$-smooth functions, smoothness on all lines, converse Taylor theorem, strongly $\alpha(\cdot)$-paraconvex functions.

MSC: 26B25; 26B35.