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q-Moment Measures and Applications: a New Approach via Optimal Transport

In 2017, Bo'az Klartag obtained a new result in differential geometry on the existence of affine hemisphere of elliptic type. In his approach, a surface is associated with every convex function $\varphi\colon \mathbb{R}^n \to (0, +\infty)$ and the condition for the surface to be an affine hemisphere involves the 2-moment measure of φ (a particular case of q-moment measures, i.e measures of the form $(\nabla\varphi)_\# \varphi^{-(n+q)}$ for $q > 0$). In Klartag's paper, q-moment measures are studied through a variational method requiring to minimize a functional among convex functions, which is achieved using the Borell-Brascamp-Lieb inequality. In this paper, we attack the same problem through an optimal transport approach, since the convex function φ is a Kantorovich potential (as already done for moment measures in a previous paper). The variational problem in this new approach becomes the minimization of a local functional and a transport cost among probability measures ϱ and the optimizer ϱ_{opt} turns out to be of the form $\varrho_{\text{opt}} = \varphi^{-(n+q)}$.

Keywords: Affine spheres, convex functions, Wasserstein spaces.

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