Journal of Convex Analysis 28 (2021) 803–818

T. Banakh
I. Franko National University, Lviv, Ukraine
and: J. Kochanowski University, Kielce, Poland
t.o.banakh@gmail.com

S. Gabriyelyan
Department of Mathematics, Ben-Gurion University of the Negev, Beer-Sheva, Israel
saak@math.bgu.ac.il

Locally Convex Properties of Baire Type Function Spaces

For an infinite Tychonoff space X, a nonzero countable ordinal α and a locally convex space E over the field $\mathbb{F}$ of real or complex numbers, we denote by $B_\alpha(X,E)$ the class of Baire-α functions from X to E. In terms of the space E we characterize the space $B_\alpha(X,E)$ satisfying various weak barrelledness conditions, (DF)-type properties, the Grothendieck property, or Dunford-Pettis type properties. We solve Banach-Mazur's separable quotient problem for $B_\alpha(X,E)$ in a strong form: $B_\alpha(X,E)$ contains a complemented subspace isomorphic to $\mathbb{F}^{\mathbb{N}}$. Applying our results to the case when X is metrizable and $E=\mathbb{R}$, we show that the space $B_\alpha(X) := B_\alpha(X,\mathbb{R})$ is Baire-like (and hence barrelled), has the Grothendieck property and the Dunford-Pettis property. Further, the space $B_\alpha(X)$ is (semi-)Montel iff it is (semi-)reflexive iff it is (quasi-)complete iff $B_\alpha(X)=\mathbb{R}^X$ (for $\alpha=1$ the last equality is equivalent to X of being a Q-space).

Keywords: Baire type function spaces, Baire-like, weak barrelledness, Grothendieck property, Dunford-Pettis property, quasi-DF-space, semi-reflexive, semi-Montel.

MSC: 46A03; 46A08, 54C35.