T. Banakh  
I. Franko National University, Lviv, Ukraine  
and: J. Kochanowski University, Kielce, Poland  
t.o.banakh@gmail.com  

S. Gabriyelyan  
Department of Mathematics, Ben-Gurion University of the Negev, Beer-Sheva, Israel  
saak@math.bgu.ac.il  

Locally Convex Properties of Baire Type Function Spaces  

For an infinite Tychonoff space $X$, a nonzero countable ordinal $\alpha$ and a locally convex space $E$ over the field $F$ of real or complex numbers, we denote by $B_\alpha(X, E)$ the class of Baire-$\alpha$ functions from $X$ to $E$. In terms of the space $E$ we characterize the space $B_\alpha(X, E)$ satisfying various weak barrelledness conditions, $(DF)$-type properties, the Grothendieck property, or Dunford-Pettis type properties. We solve Banach-Mazur’s separable quotient problem for $B_\alpha(X, E)$ in a strong form: $B_\alpha(X, E)$ contains a complemented subspace isomorphic to $F^\mathbb{N}$. Applying our results to the case when $X$ is metrizable and $E = \mathbb{R}$, we show that the space $B_\alpha(X) := B_\alpha(X, \mathbb{R})$ is Baire-like (and hence barrelled), has the Grothendieck property and the Dunford-Pettis property. Further, the space $B_\alpha(X)$ is (semi-)Montel iff it is (semi-)reflexive iff it is (quasi-)complete iff $B_\alpha(X) = \mathbb{R}^X$ (for $\alpha = 1$ the last equality is equivalent to $X$ of being a $Q$-space).

Keywords: Baire type function spaces, Baire-like, weak barrelledness, Grothendieck property, Dunford-Pettis property, quasi-DF-space, semi-reflexive, semi-Montel.

MSC: 46A03; 46A08, 54C35.